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Investment and Risk Analysis Applied to the Petroleum Industry

R.Motta, UFRJ, and G.Caloba, UFRJ, and L. Almeida, UFRJ, and A. Moreira, UFRJ, and M. Nogueira, UFRJ, and L. Cardoso, UFRJ, and L. Berlink, UFRJ

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Abstract

In Brazil, for the last four decades, Petrobras, a state-owned company, had retained a monopolistic concession for the exploration of Oil & Gas prospects, when, in 1998, the law was changed and a broad range of opportunities were created. Since then, Petrobras has been offering a series of Joint Venture opportunities for foreign oil companies to farm in.

Presently, a number of international oil companies installed offices locally and started to participate in the upstream sector. ANP, the National Petroleum Agency, is offering for tender various blocks both offshore and onshore. The first bid round was a major success, in mid 1999, the second bid round being scheduled for the third quarter of year 2,000.

Blocks leased in the first bid round are now being explored by means of 2D and 3D seismic, and the first exploratory wells will be soon drilled.

This article presents a framework to perform an economic evaluation of oil & gas prospects, taking into account the risk factor. The Net Present Value of each block is first computed based upon an average value. Following that, Monte Carlo simulation is used to compute the downside risk. The inputs are determined by means of probability distributions. Reservoir parameters, capital and operating costs and oil prices are fed into the program, revealing the NPV sensitivity to each factor.

The problem is further structured by means of decision trees. The Expected Monetary Value, based upon perceived probabilities of success, is estimated, utility functions being used to account for risk aversion. The optimum level of participation in each prospect is determined, using the so-called Certainty Equivalent concept. Finally, the

formation of an optimum portfolio is discussed, so that any oil company can establish its strategy in joint bids and farm ins.

Four hypothetical prospects are evaluated; with different levels of investment and risk profiles, namely, from higher probability of success to new frontier, lower probability of success. The correspondence with real life may be, respectively, the Campos Basin, where 70% of Brazilian oil production comes from, and basins like Pelotas, where there are no producing wells, with little seismic available.

The framework suggested in this work could be useful for any company that is currently involved with exploration activities. Additionally, it may be used for analyzing any investment with a certain degree of uncertainty or volatility.

Introduction

The contemporary Brazilian scenario offers opportunities of investments to foreign petroleum companies, normally in the upstream sector that until recently, was restricted to Petrobras.

This article shows the methodology to take decisions that involve risk. When there is no such thing as a variety of results, uncertainty or risk, the decision criteria can be based upon the net present value only.

We will work on an example that has four different prospects, with different internal rates of return and four standard deviations for these rates. Prospect A has an internal rate of return of 23% per annum, and a standard deviation of 13% per year. Prospect B has an IRR of 29% and deviation of 24%. The alternative C has an IRR of 37%, and a standard deviation of 41% and finally, prospect D has an IRR of 19% and a deviation of 7% per annum. The probability of success for each prospect also varies.

We will analyze these different options, coming to some results that will be a basis for building up the optimized investment portfolio.

Geological Risks:

The main factors and mechanisms that control petroleum accumulations are:

- Existence of trap
- Source rock
- Thermal maturation

- Migration and timing
- Reservoir (storage capacity)
- Seal
- Productivity

To each of these events, are probabilities of success and failure assigned, based upon vague or registered experiences. As there is a lack of significant data based on which inferences of future risks may be made, the vague experience (or professional experience) is strongly used in the determination of the probabilities of the occurrence of the events.

The probability of existence of oil in a given place may be described as the product of all the probabilities of each event occurring individually.

$$PS = G * E * R * S * M * T \dots\dots\dots(1)$$

Where:

- PS = Success Probability;
- G = Generator
- E = Structural or Stratigraphic trap
- R = Reservoir;
- S = Seal;
- M = Migration; and
- T = Timing.

Economic Risks

These risks are established from the analysis of the parameters, which determine the size distribution (area and volume) of the possible accumulations of oil (structure area, thickness, porosity, oil saturation), the indices of success resulting in probabilities of finding fields of different sizes.

The volume of recoverable oil is given by the formula:

$$V = A * E * \Phi (1 - S_w) / B_o \dots\dots\dots(2)$$

Where:

- A = Accumulation area;
- E = Reservoir thickness;
- Φ = Effective reservoir porosity;
- S_o = Oil saturation; and
- R = Recovery Factor.

It is possible to calculate this volume through an EXCEL worksheet, with hypothetical data, by two different means:

- Deterministically, with expected values or means, resulting in a constant value; and
- With probability distributions for the above variables (figures 2,3,4), resulting in different values for V.

In the first alternative, the calculation is made with the best possible estimation of the components of the formula.

Nothing can assure that the calculated number is going to be right, for there is an uncertainty variation in the estimated parameters.

In the second case, some cases can be taken from the curve of accumulative frequency, as the pessimistic, the realistic and the optimistic. Those could be the reserve values for the 10%, 50% and 90% percentiles, that is, a number that leaves only 10% of the cases below (pessimistic); 50% (realistic); and 90% (optimistic).

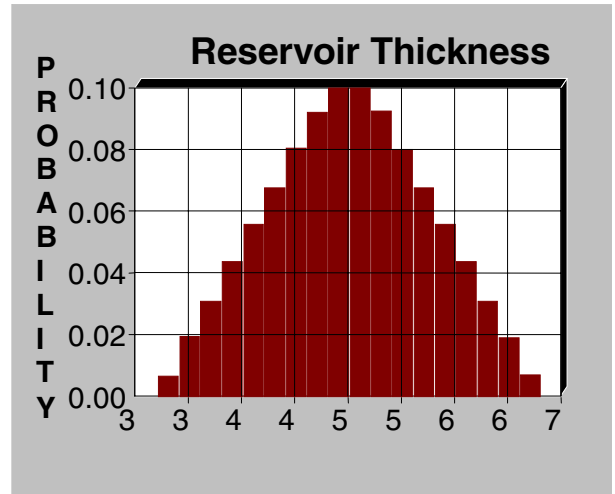


FIGURE 1 – Reservoir thickness probability distribution

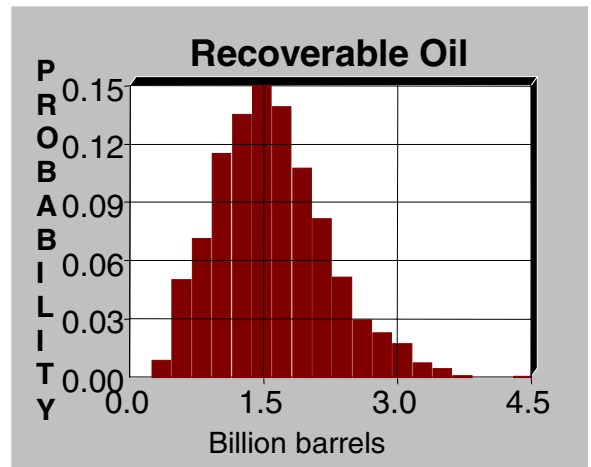


FIGURE 2 – Recoverable oil probability distribution

Production Forecasting Risks

Once the recoverable reserves are established, we need to estimate how fast the production, or the exploitation, or the depletion of the reserves of oil and gas will take place. Some important factors for these items are:

- Number of wells;
- Percentage of dry holes or success ratio (*);
- Drainage area or recovery per well;
- Productivity index per well;
- Operating constraints on production rates;

Initial decline rates;
Abandonment rates or other abandonment conditions;
and product prices;

(*) Note: In the 1980's, with the advent of 3D seismic surveys, the exploration risks were strongly reduced. The probability of success in drilling was raised from 40% to approximately 60%.

One of the most commonly used formulas for production forecasting is the exponential decline curve:

$$q = q_i e^{(-at)} \dots\dots\dots(3)$$

This deterministic model can be converted in a stochastic or probabilistic model, treating the two parameters, q_i and a , which represent respectively, initial production and rate of decline, as random variables instead of adopting constant values. The time, in years, is given by the variable t , and the production in each year, q , is declining year after year.

Figure 3 shows a range of values for the production, which declines exponentially with time. The upper limit leaves 90% of the cases below it; the central one, 50%; and the lower limit keeps only 10% of the cases below. These are the three scenarios mentioned earlier in this article.

To obtain the results simulated in the graphic, the rate of decline (a) used, as entry data, was a normal distribution with mean value of 10% per year, and standard deviation of 0.5%. The initial production value (q_i) is the result of a spreadsheet previously calculated, and is already a random variable. The simulated outputs of one spreadsheet are the inputs of another one.

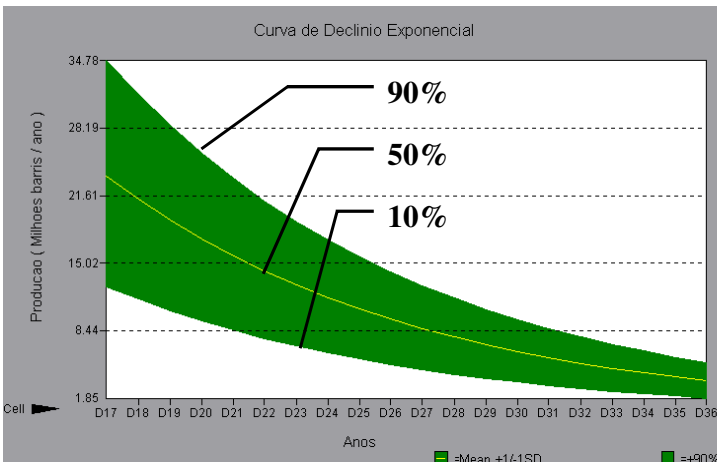


FIGURE 3- Exponential decline curve

Figure 4 shows US\$ capital expenditures (CAPEX) invested in exploration (E), Appraisal (A) and development (D), along the years. A number of wells are drilled, and the Revenue curve starts with production, which in turn generates operational expenditures (OPEX).

The level of production grows up fast, during the early years, then stabilizes for 3 to 4 years, and then decreases following the decline curve.

There is an economic cut point, when revenues break-even with costs. Finally, there are abandonment costs, due to environmental reasons.

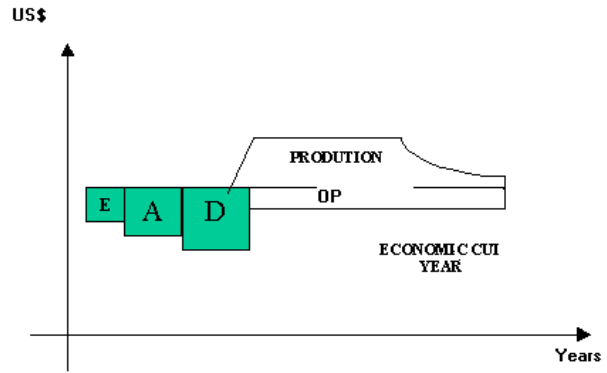


FIGURE 4- Investment and income during E&P phases of n oil wells

Cash Flow

The cash flow of a typical upstream investment is shown on the table 2, in the annex.

The phases of exploration, appraisal and development may succeed, resulting in the continuation of the production. There are risks, however, of an early abortion in these phases, generating only losses (E, in figure 5, for instance).

Decision Trees

In decision-making, it is common to use decision trees, which consist in a graphic representation of the options and all its consequences, in an organized and clear way, simplifying the work of the decision-maker. To design the tree, it is necessary to list the options and quantify them as much as possible. One example of a simplified tree is in figure 5.

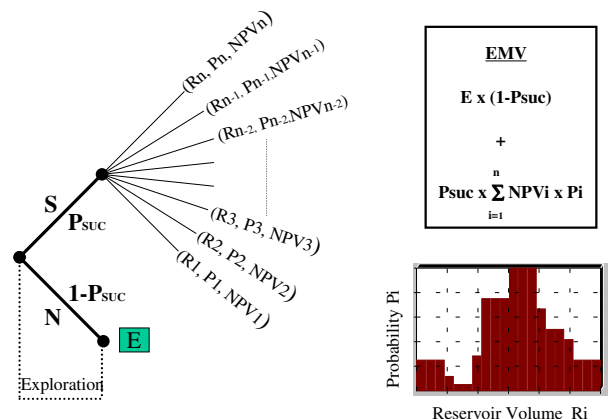


FIGURE 5- Simplified Risk Tree

Expected Monetary Value (EMV)

The expected monetary value is a weighted average of profits and losses multiplied by their probabilities.

Mathematically speaking, it is given by the expression:

$$VME = p \cdot NPV1 + (1-p) \cdot NPV2 \dots \dots \dots (4)$$

Where NPV1 and NPV2 would be the Net Present Values of cash flows with success and loss in the exploration. Obviously, there would be many other options for the flows (bigger or smaller amount, variability in the costs).

The EMV method does not reduce or eliminate the risks but is a tool to make better evaluation, understanding and quantification of the risks to which the company may be exposed. In most cases, it is very complex to define the best project through the EMV. This tool is generally used in companies that do not have capital restriction. The EMV does not show the projects probability of success; neither does it reproduce the human behavior in risk situations.

Preference Theory: Utility Functions

One way to express the preference in relation to risk activities is through the Utility Functions that are constructed giving upper values to better results of an investment, and lower values to worse results. With the intermediate values, it is possible to plot the value of the investment *versus* utility function value. This curve is a parable with concavity down, in the case of risk-averted managers, a straight line to the managers indifferent to the risk and a parable with upper concavity to risk-taking managers.

This function can be defined empirically or analytically. The empirical part can be done by means of an interview, the analytic, using a linear, exponential, logarithmic or square root function.

To apply the preference theory method, two concepts are important: the certainty equivalent and the function of risk-aversion associated with the utility function. The certainty equivalent is the value in money that an investor would take, without risk, as a return for a risky choice, that is, a comparison between risky and not-risky investments.

The exponential utility function (the only that shows constant risk-aversion) may be used as substitute for other utility functions under some conditions, and this function is very important in valuation and implementation of risk analysis in petroleum exploration projects.

This function is defined as $U(x) = 1 - e^{-c \cdot x}$, where c is the risk-aversion coefficient of the decision-maker and x represents the participation of the company in the project, in percentage.

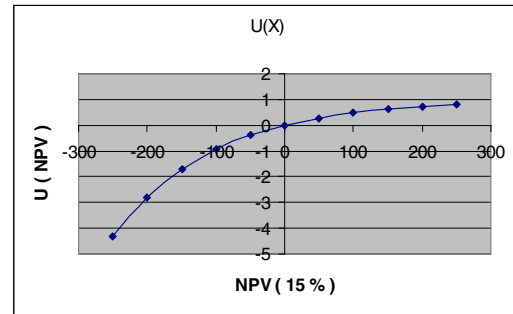


FIGURE 6-Utility Function

Certainty equivalent

Sometimes, a risky option can be compared with one that does not imply risk. The Executives may ask himself: What would be the lowest amount of money that I would accept, without risk, as a return for the risky option with a given expected monetary value? This value is the certainty equivalent of the risky choice.

For those who do not like risk, the certainty equivalent is most of the times, lower than EMV. The difference between the certainty equivalent and the EMV is the "Risk Premium": the money value of the uncertainty as it is perceived by whom ever is willing to take it. The formula for the certainty equivalent is:

$$EqC(X) = - \frac{1}{c} * \ln [p_1 e^{-c(x)VP_{L1}} + p_2 e^{-c(x)VP_{L2}}] \dots (5)$$

Where:

c is the level of risk aversion of the decision-maker;

EqC = Certainty Equivalent of the project in million dollars;

pi = Probability of success of the event i;

NPV1 = Net present value of event 1 (Success);

NPV2 = Net present value of event 2 (Failure);

c = Coefficient of risk aversion (= 1/ R ; R = risk tolerance)

x = Level of monetary participation of the company in the project (%)

The relation between these concepts, utility function and certainty equivalent is better understood when we are willing to incorporate the utility functions in a model. In order to do so we need to follow the steps below:

- 1) Use the utility function to calculate the utility of each possible result.
- 2) Calculate the expected utility for the specific example. In our case, we will use example A.
- 3) To convert the expected utility in the certainty equivalent, we use the inverse of the utility function.

That is to say, the certainty equivalent is obtained through the inverse function of the utility function. This idea will be better understood with an example.

In example A, we have:

$$NPV1 = 228.42$$

$$NPV2 = -160.00$$

Using $R=150$ and replacing in the formula:

$$U(x) = 1 - e^{-c(x)} \dots\dots\dots(6)$$

We get:

$$U(X) = 0.78 \text{ (for } NPV1 = 228.42) \text{ and}$$

$$= -1.90 \text{ (for } NPV2 = -160.00)$$

Therefore the expected utility of our example is -0.29.

If we compute the inverse function of the utility function (using the obtained value (-0.29)), the value for the certainty equivalent of the example A is finally obtained.

Portfolio Selection

We will take as an example the investments A, B, C and D, consisting of the exploration (E), development and production phases, with investment, revenue and operational costs (R/O). The internal rates of return of these prospects and their standard deviations were already described in the introduction. As a matter of simplicity, instead of using cash flows 30-year long, the ones shown here will be only 5-year long (table 1 in the Annex).

Such cash flows are shown in deterministic and probabilistic forms. The simulated results were generated through a Monte Carlo simulation software, an Add-in to spreadsheet models.

The final objective of this work is to provide a guidance for investment selection, according to the risk profile (given by the utility function) and the risk tolerance (given by R, in million of dollars). An empirical study in the United States, analyzing 50 independent petroleum companies from 1981 to 1990, concluded that the risk tolerance is generally 1/4 of the exploration capital of the company.

In our example, we adopted $R= US\$ 150$ million, and an exponential utility function, as described earlier (figure 7 in annex).

Following the criteria of maximizing the Net Present Value in the decisions, prospect A is selected through the Decision Tree software. Note the word TRUE in branch A and an EMV of US\$ 73 million, approximately, the biggest amongst the branches A, B, C and D. So, the decision is to invest and invest in the A prospect. This decision results from an analysis that ignores risk.

In figure 8 (in annex), we have a decision tree with the Certainty Equivalents calculated for $R=US\$ 150$ million. Note that the D prospect was selected, for it maximizes the Certainty Equivalent, with a value of US\$ 14 million, approximately.

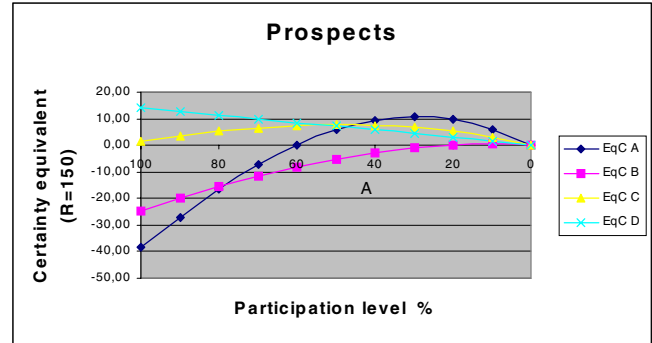


FIGURE 9- Variations of the certainty equivalent (R=150)

In figure 9, the variation of the certainty equivalent for each level of investment participation in the prospects A, B, C and D (from 0 to 100%) is shown (note that the x axis grows from right to left). R is constant, and has a value of US\$ 150 million. It is implicit that the prospects allow any percentage share of any company. In other words, companies can do joint ventures to invest in the prospects.

Note that the best participation (shares) are those maximizing the certainty equivalent, for each prospect. In the example, for the prospect A, the best share is 30%; for B, 10%; for C, 50%; and for the prospect D, 100%.

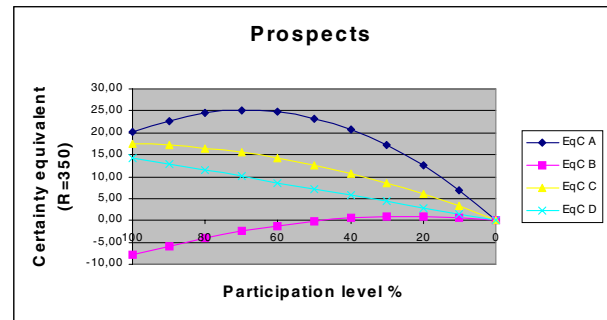


FIGURE 10- The variation of the certainty equivalent (R=350)

In figure 10 there is another example of variations of the certainty equivalent, in this case with a risk tolerance of US\$ 350 million. The shares for each prospect A, B, C and D vary from 0% to 100% (note that the x-axis grows from right to left, again).

In this second example, the best share for each prospect is for A, 70%; for B, 20%; for C, 100%; and for D, 100%. Comparing the two examples, it is possible to understand how the best shares are established according to risk tolerance.

As an example, a smaller oil company X can take a 30 % share in prospect A, given a risk tolerance of US\$ 150 million, and a bigger company Y can take the balance – a 70% share- ,in prospect A, with a \$300 million risk tolerance.

Conclusion

The methodology described in this article uses the most modern software tools available, and provides answers to the uncertain problems. Many companies penalize projects using a basic discount rate, generally equal to the capital cost (WACC-*Weighted Average Cost of Capital*), increased by a premium for the risk (*spread*). This discount rate is also known as *Hurdle Rate* and establishes an exceedingly heavy cash flow penalty in long-term investments, as those of the petroleum industry in the *upstream sector*, which can easily last for 30 years or more.

The approach proposed assumes a basic discount rate, without any *spread* increase. Risk treatment will be done using the utility function and its inverse: the Certainty Equivalent.

The problem could still be extended by means of a sensitivity analysis of the parameters, or through the use of linear programming to study all the portfolio possibilities with different risk tolerance coefficients (R), or even by genetic algorithms.

For ANP's second bid round, this methodology could be very handy for petroleum companies.

With the objective of preparing human resources for the petroleum industry, these concepts have been presented in the undergraduate Production Engineering Course of the School of Engineering at UFRJ and in the *Master of Business in Petroleum* course of COPPE/UFRJ.

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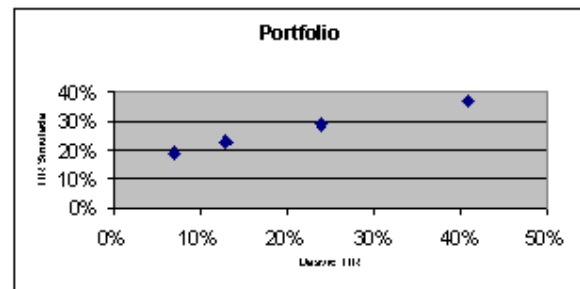
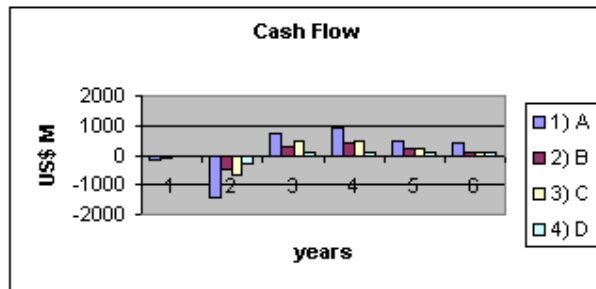
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TABLE 1-Upstream Cash Flow (five years)

Prospect	E	R/O					TIR	TIR Simul	Desvio TIR	NPV(15%)
1) A	-160	-1400	700	900	500	400	23%	23%	13%	\$228,42
2) B	-80	-500	300	400	200	100	29%	29%	24%	\$139,14
3) C	-40	-680	500	450	200	100	34%	37%	41%	\$206,72
4) D	-20	-270	110	110	110	110	18%	19%	7%	\$18,30

Prospect	NPV(15%)	NPV(15%)	NPV(15%)
	Expl	Prod	E&P
1) A	(\$160,00)	\$388,42	\$228,42
2) B	(\$80,00)	\$219,14	\$139,14
3) C	(\$40,00)	\$246,72	\$206,72
4) D	(\$20,00)	\$38,30	\$18,30



Prospect	Expected Value NPV(15%)	Scenario		
		10% NPV(15%)	50% NPV(15%)	90% NPV(15%)
1) A	228	-208	203	689
2) B	139	-135	102	467
3) C	206	-240	127	749
4) D	18	-32	19	68
	Faixa	20%	60%	20%

Prospect	Downside Risk	Risk Factor
1) A	27%	1: 3,70
2) B	31%	1: 3,23
3) C	36%	1: 2,78
4) D	33%	1: 3,03

TABLE 2 Simplified Cashflow for Petroleum Exploration Projects

Year	Oil Price \$/bbl	Oil Production MM bbl	Investment			OPEX	Gross	Royalties	Cumulative	Government Take		Overhead	Income	Before Tax	After Tax Cashflow	
			Exploration	Appraisal	Development	MM\$	Revenues	15%	Revenues	%	US\$	MM\$	Tax	Cashflow	Yearly	Cumulative
1998	15,0	0	5,4	0	0	0	0	0	0	0,0%	1,5	0,00	0,0	-6,90	-6,90	-6,90
1999	15,0	0	6,3	0	0	0	0	0	0	0,0%	2,0	0,00	0,0	-8,30	-8,30	-15,20
2000	15,0	0	10,4	0	0	0	0	0	0	0,0%	2,0	0,00	0,0	-12,40	-12,40	-27,60
2001	15,0	0	0	0	0	0	0	0	0	0,0%	0,0	2,00	0,0	-2,00	-2,00	-29,60
2002	15,0	0	0	4,6	0	0	0	0	0	0,0%	0,0	2,00	0,0	-6,60	-6,60	-36,20
2003	15,0	0	0	9,6	0	0	0	0	0	0,0%	0,0	2,00	0,0	-11,60	-11,60	-47,80
2004	15,0	0	0	0	45,3	0	0	0	0	0,0%	0,0	2,00	0,0	-47,30	-47,30	-95,10
2005	15,0	0	0	0	144,4	0	0	0	0	0,0%	0,0	2,50	0,0	-146,90	-146,90	-242,00
2006	15,0	8,5	0	0	122,7	16,8	127,5	19,1	128	0,0%	0,0	3,15	59,0	-34,28	-93,25	-335,25
2007	15,0	12,7	0	0	68,1	22,2	190,5	28,6	318	0,0%	0,0	3,15	91,1	68,48	-22,62	-357,88
2008	15,0	17,3	0	0	68,1	28,7	259,5	38,9	578	0,0%	0,0	3,15	125,9	120,63	-5,25	-363,13
2009	15,0	20,2	0	0	66,8	33,6	303	45,5	881	0,0%	0,0	3,15	147,3	154,00	6,73	-356,40
2010	15,0	26,4	0	0	66,8	38,1	396	59,4	1277	0,0%	0,0	3,15	197,0	228,55	31,55	-324,85
2011	15,0	31,1	0	0	32,9	41,3	466,5	70,0	1743	6,4%	20,6	3,15	221,1	298,55	77,47	-247,38
2012	15,0	28,5	0	0	0	40,2	427,5	64,1	2171	6,3%	20,4	3,15	199,9	299,66	99,79	-147,59
2013	15,0	25,6	0	0	0	37,2	384	57,6	2555	6,4%	18,5	3,15	178,4	267,54	89,09	-58,50
2014	15,0	23,4	0	0	0	35,9	351	52,7	2906	6,6%	17,3	3,15	161,4	241,98	80,58	22,08
2015	15,0	21,9	0	0	0	33,6	328,5	49,3	3234	6,7%	16,5	3,15	150,8	226,02	75,26	97,34
2016	15,0	19,7	0	0	0	31,3	295,5	44,3	3530	7,0%	15,4	3,15	134,3	201,33	67,04	164,39
2017	15,0	17,4	0	0	0	30,2	261	39,2	3791	7,2%	13,8	3,15	116,5	174,70	58,18	222,56
2018	15,0	16,8	0	0	0	28,1	252	37,8	4043	7,6%	14,1	3,15	112,6	168,81	56,21	278,77
2019	15,0	15,1	0	0	0	27,2	226,5	34,0	4269	7,9%	13,1	3,15	99,5	149,11	49,66	328,43
2020	15,0	13,6	0	0	0	25,9	204	30,6	4473	8,2%	12,1	3,15	88,2	132,26	44,04	372,47
2021	15,0	12,9	0	0	0	24,4	193,5	29,0	4667	8,2%	11,5	3,15	83,7	125,44	41,77	414,24
2022	15,0	11,7	0	0	0	23,3	175,5	26,3	4842	7,4%	9,3	3,15	75,6	113,41	37,77	452,01
2023	15,0	10,2	0	0	0	22,7	153	23,0	4995	6,8%	7,3	3,15	64,6	96,90	32,27	484,28
2024	15,0	9,5	0	0	0	21,8	142,5	21,4	5138	6,2%	6,2	3,15	60,0	90,02	29,98	514,25
2025	15,0	8,7	0	0	0	20,9	130,5	19,6	5268	5,6%	5,0	3,15	54,6	81,83	27,25	541,50
2026	15,0	8,3	0	0	0	19,7	124,5	18,7	5393	5,1%	4,4	3,15	52,4	78,58	26,17	567,67
2027	15,0	6,4	0	0	0	17,3	96	14,4	5489	3,8%	2,4	3,15	39,2	58,71	19,55	587,22
2028	15,0	4,8	0	0	0	13,8	72	10,8	5561	3,1%	1,5	3,15	28,5	42,78	14,25	601,46
TOTAL		370,7	22,1	14,2	615,1	634,2	5560,5	834,075			214,9	82,95	2541,5	3143,00	601,46	

NPV@								
Per annum	7,50%	R\$ 115,99	R\$ 20,26	R\$ 10,13	R\$ 324,80	R\$ 189,56	R\$ 1.739,81	R\$ 260,97
Discount	10%	R\$ 82,46	R\$ 19,72	R\$ 9,10	R\$ 266,28	R\$ 133,61	R\$ 1.236,93	R\$ 185,54
Rate	12,50%	R\$ 59,74	R\$ 19,22	R\$ 8,20	R\$ 219,70	R\$ 96,22	R\$ 896,13	R\$ 134,42

R\$ 27,82	R\$ 808,90	R\$ 845,16	R\$ 36,26
R\$ 20,88	R\$ 577,56	R\$ 559,31	(R\$ 18,25)
R\$ 16,16	R\$ 419,97	R\$ 371,82	(R\$ 48,15)
		Before Tax	After Tax
		IRR	31%
		(per annum)	9%

EMV

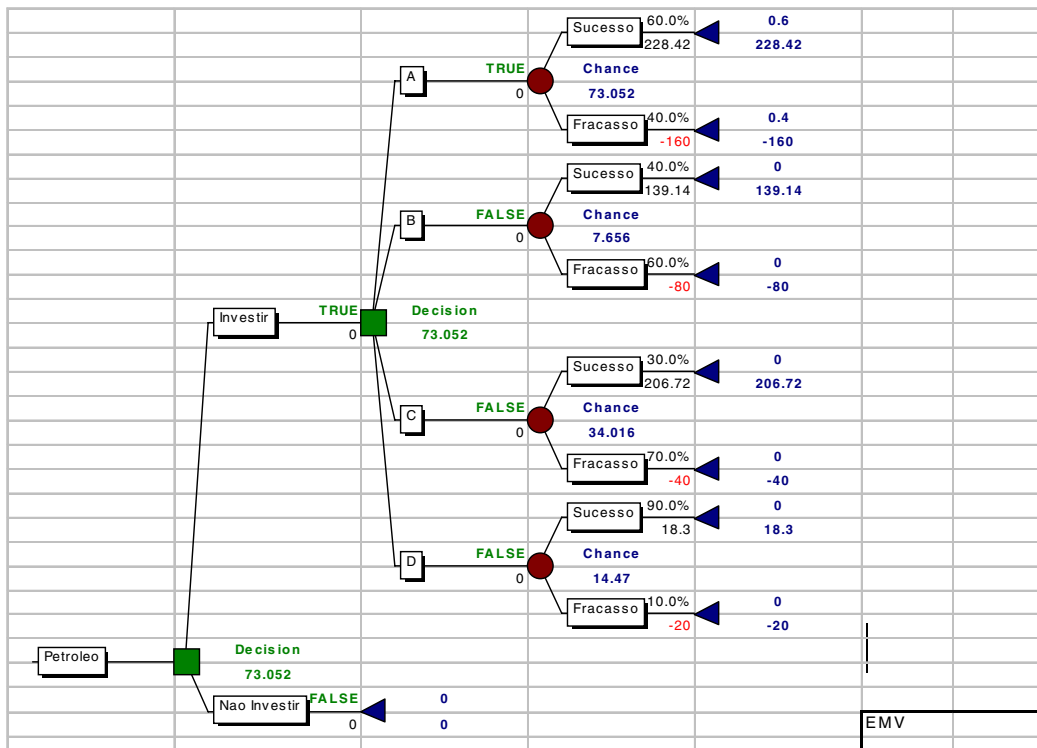


FIGURE 7-The EMV for each prospect.

Certainty Equivalents (R= 150)

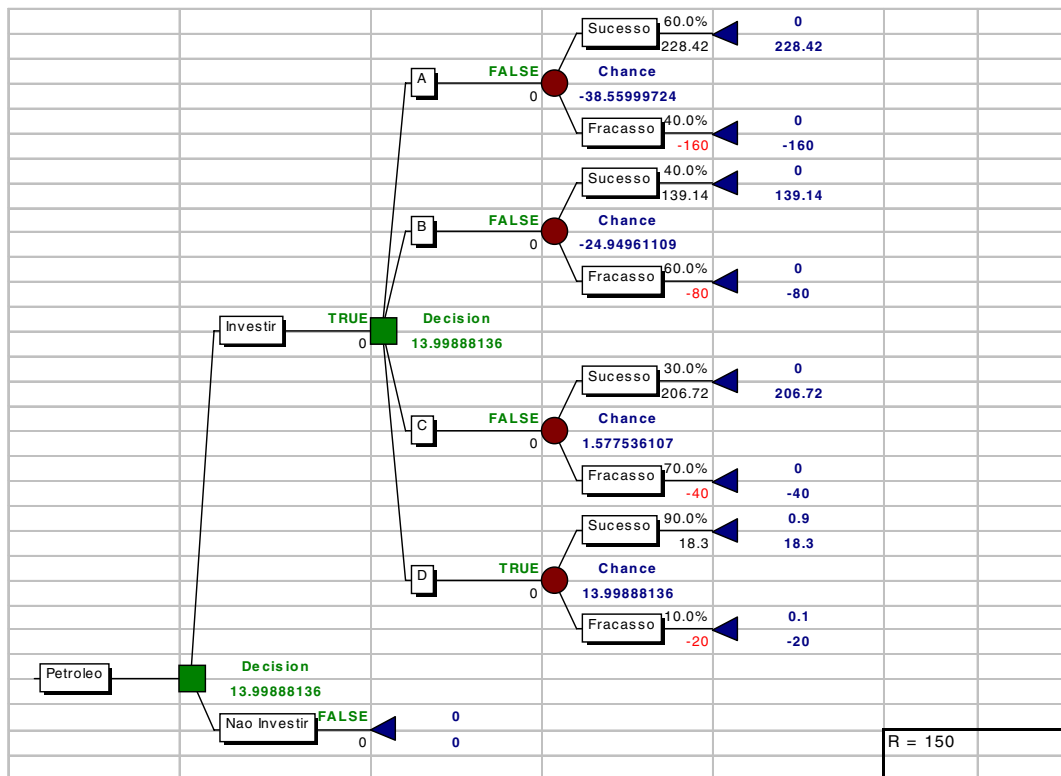


FIGURE 8- Variations of the certainty equivalent (R= 150)