Delivering Integrated, Sustainable, Water Resources Solutions

Introduction to Engineering Reliability

Robert C. Patev

North Atlantic Division – Regional Technical Specialist

(978) 318-8394

Topics

- Reliability
- Basic Principles of Reliability Analysis
  - Non-Probabilistic Methods
  - Probabilistic Methods
    - First Order Second Moment
    - Point Estimate Method
    - Monte Carlo Simulation
    - Response Surface Modeling

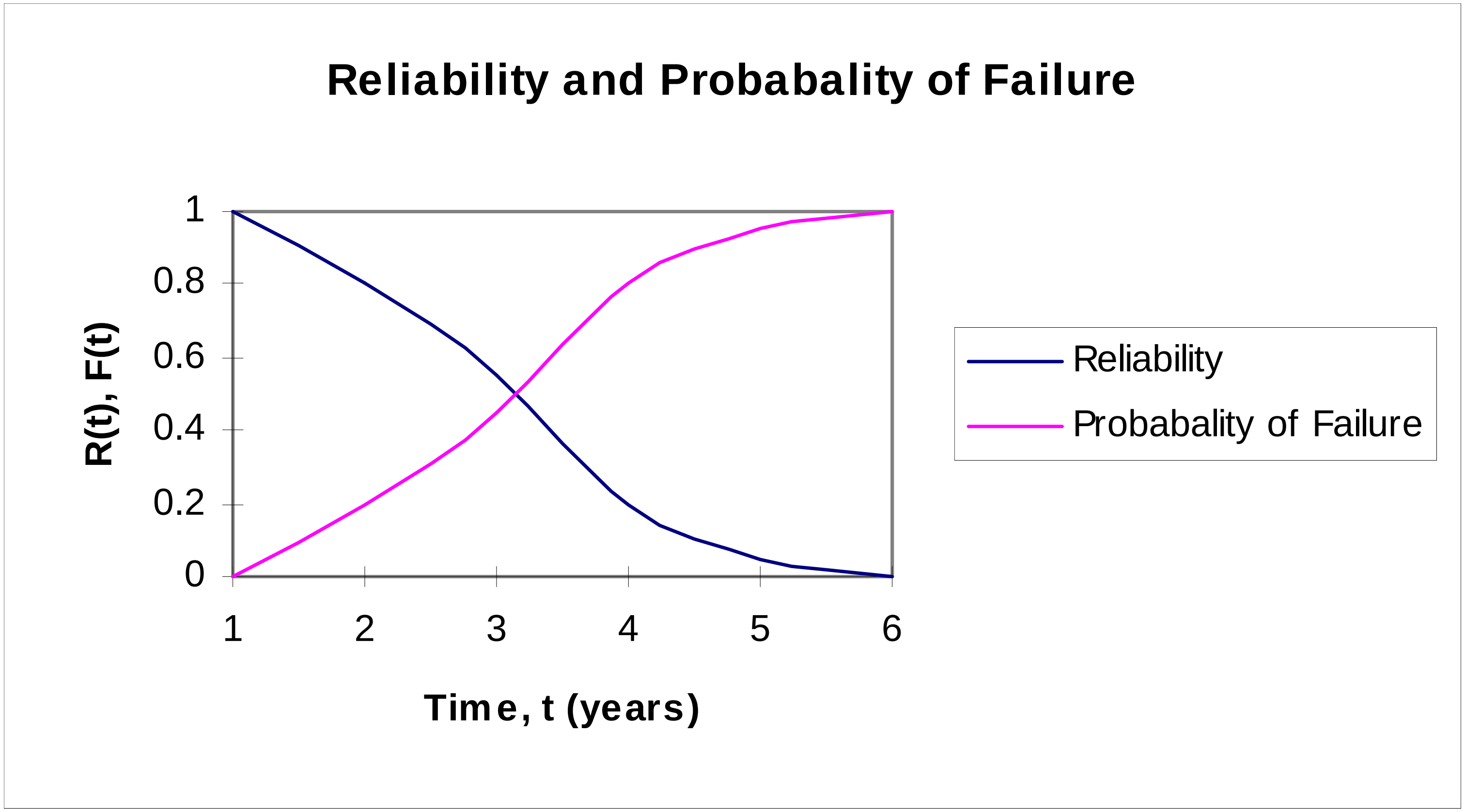
- Reliability

  – "Probability that a system will perform its intended function for a specific period of time under a given set of conditions"

$$R = 1 - P_f$$

  – Reliability is the probability that unsatisfactory performance or failure will not occur

Reliability



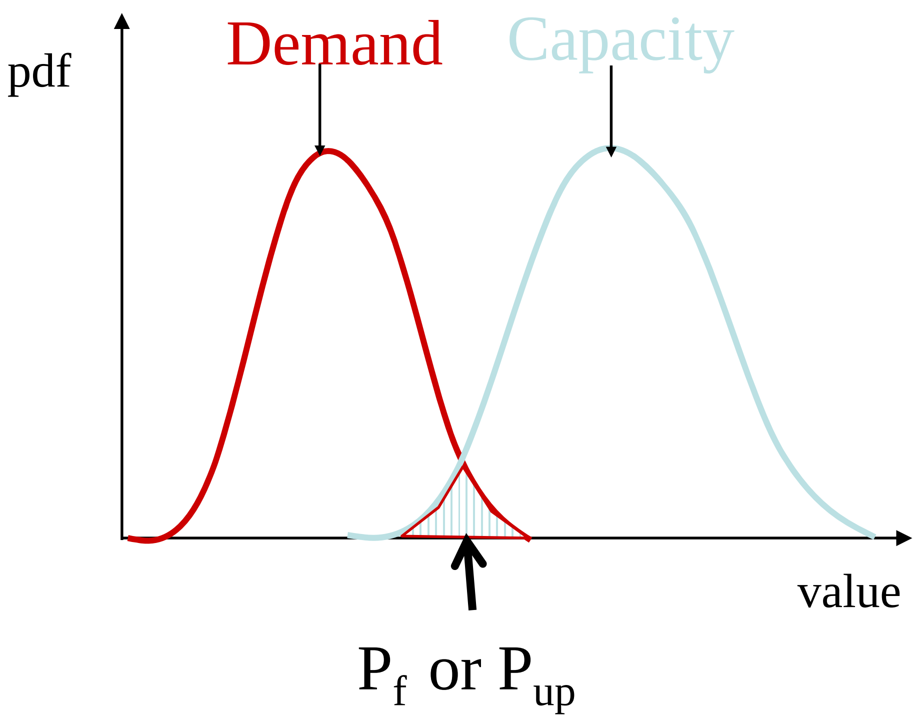
- Probability of Failure, "P_f"

  – Easily defined for recurring events and replicate components (e.g., mechanical and mechanical parts)

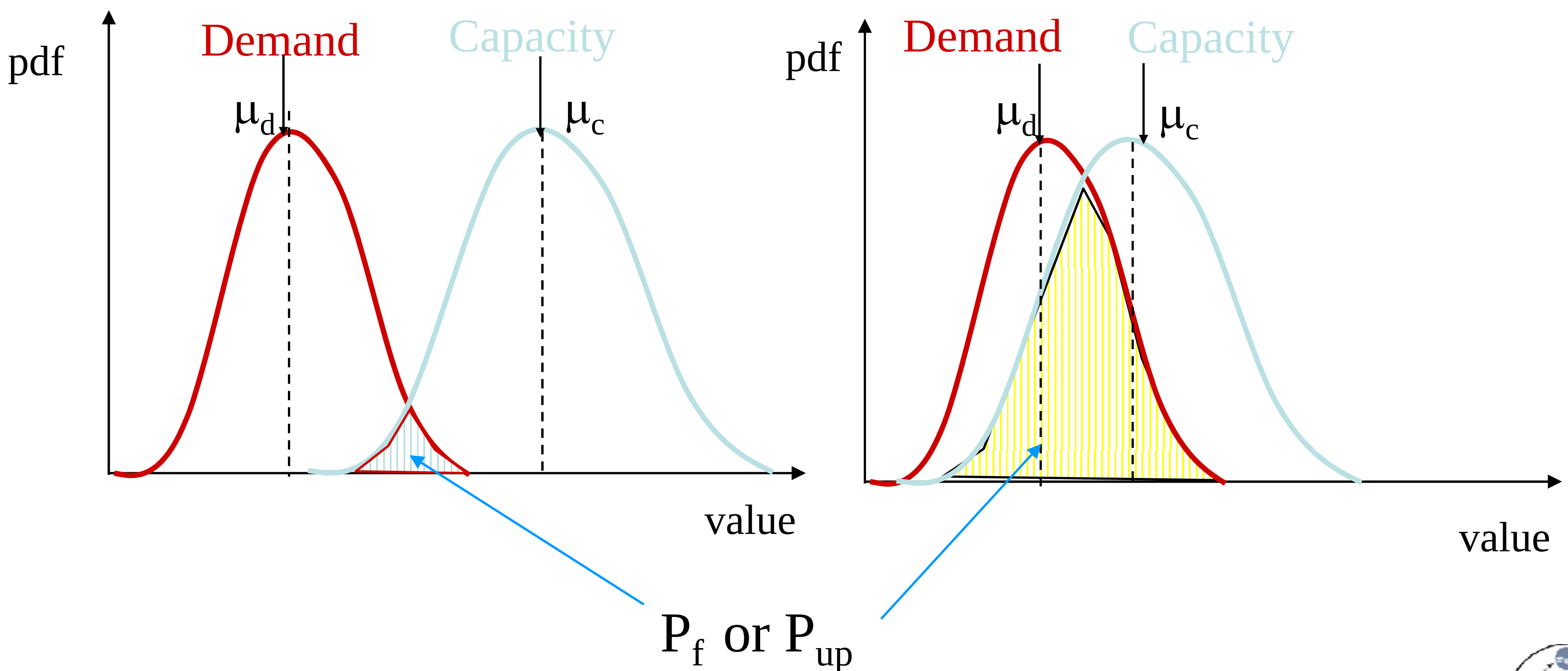
- Probability of Unsatisfactory Performance, P(u) "Pup"

  – Nearly impossible to define for non-recurring events or unique components (e.g., sliding of gravity structures)

Reliability

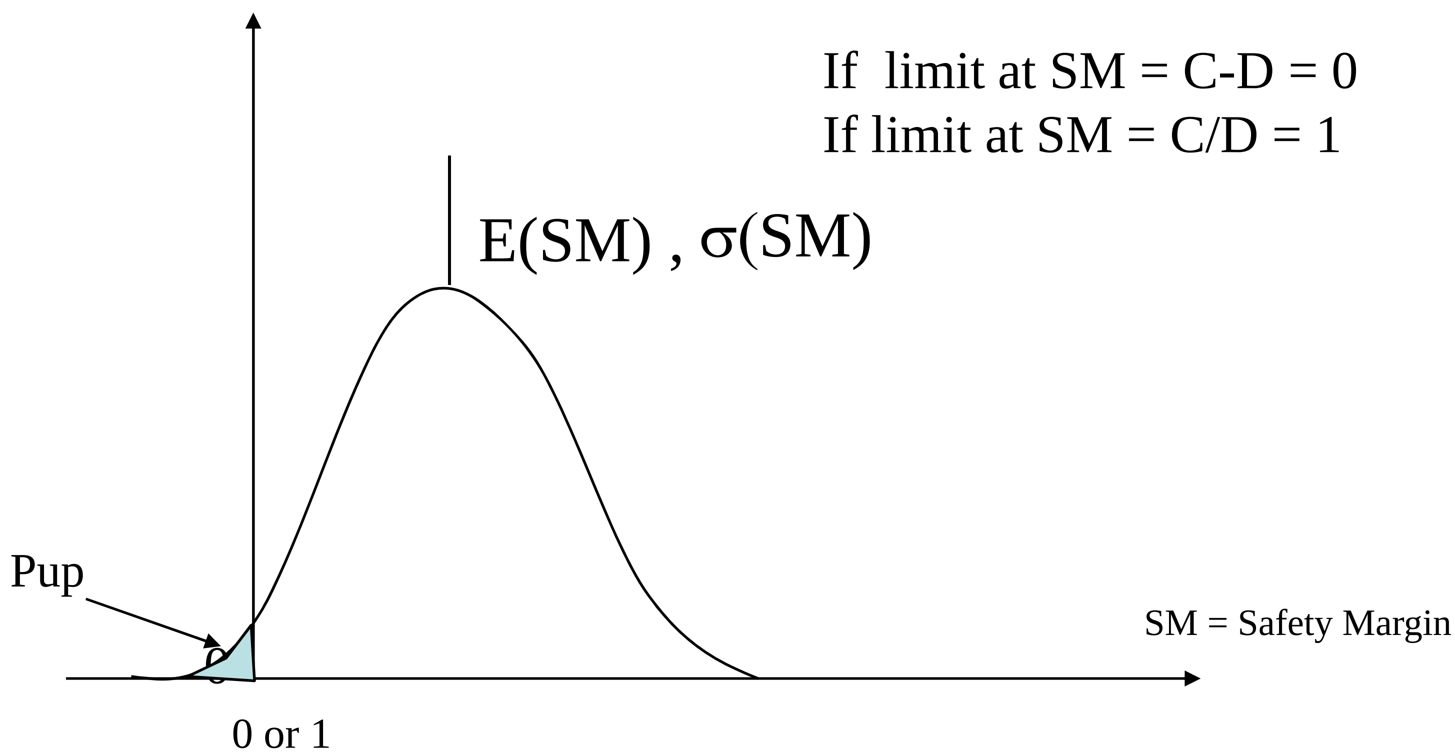


Reliability



Reliability

Safety Margins



• Basic Principles of Reliability Analysis

- Identify critical components
- Use available data from previous design and analysis
- Establish base condition for component
- Define performance modes in terms of past levels of unsatisfactory performance
- Calibrate models to experience
- Model reasonable maintenance and repair scenarios and alternatives

- <u>Non-Probabilistic Reliability Methods</u>

  – Historical Frequency of Occurrence

  – Survivorship Curves (hydropower equipment)

  – Expert Opinion Elicitation

- Probabilistic Reliability Methods
  - Reliability Index (β) Methods
    - – First Order Second Moment (Taylor Series)
    - – Advanced Second Moment (Hasofer-Lind)
    - – Point Estimate Method
  - Time-Dependent (Hazard Functions)
  - Monte Carlo Simulation
  - Response Surface Modeling

- <u>Historical Frequencies</u>

  – Use of known historical information for records at site to estimate the failure rates of various components

  – For example, if you had 5 hydraulic pumps in standby mode and each ran for 2000 hours in standby and 3 failed during standby. The failure rate during standby mode is:

Total standby hours = 5(2000 hours) = 10,000 hours

Failure rate in standby mode = 3 / 10,000

= 0.0003 failures per hour

- Manufacturers’ survivorship/mortality curves
  - Curves are available from manufacturers’ for different motors, pumps, electrical components, etc...
  - Curves are developed from field data and “failed” components
    - Caution is to be exercised on mode of failure
    - Failure data may have to be censored
  - However, usually this data id not readily available for equipment at Corps projects except mainly hydropower facilities
  - Report available at IWR on hydropower survivorship curve as well as many textbooks on the subject

- Expert Opinion Elicitation (EOE)
  - Solicitation of "experts" to assist in determining probabilities of unsatisfactory performance or rates of occurrence.
  - Need proper guidance and assistance to solicit and train the experts properly to remove all bias and dominance.
  - Should be documented well for ATR/IEPR
  - Some recent projects that used EOE
    - John Day Lock and Dam – Dam Anchorage, NWP

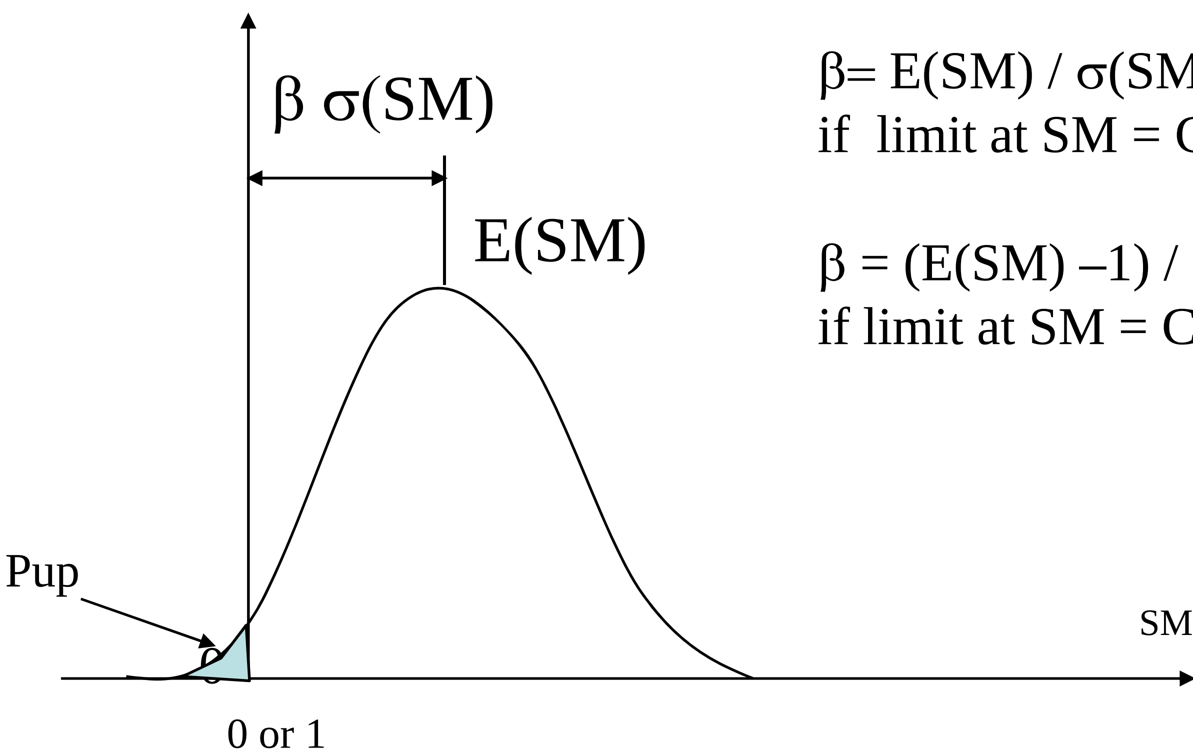
- Probabilistic Methods

  – Reliability models are:
    - defined by random variables and their underlying distributions
    - based on limit states (analytical equations) similar to those use in the design of engineering components
    - based on capacity/demand or factor of safety relationships

  – One method is the Reliability Index or β method

Reliability

β Method - Normal Distribution



$\beta = E(SM) / \sigma(SM)$
if limit at SM = C-D = 0

$\beta = (E(SM) - 1) / \sigma(SM)$
if limit at SM = C/D = 1

SM = Safety Margin

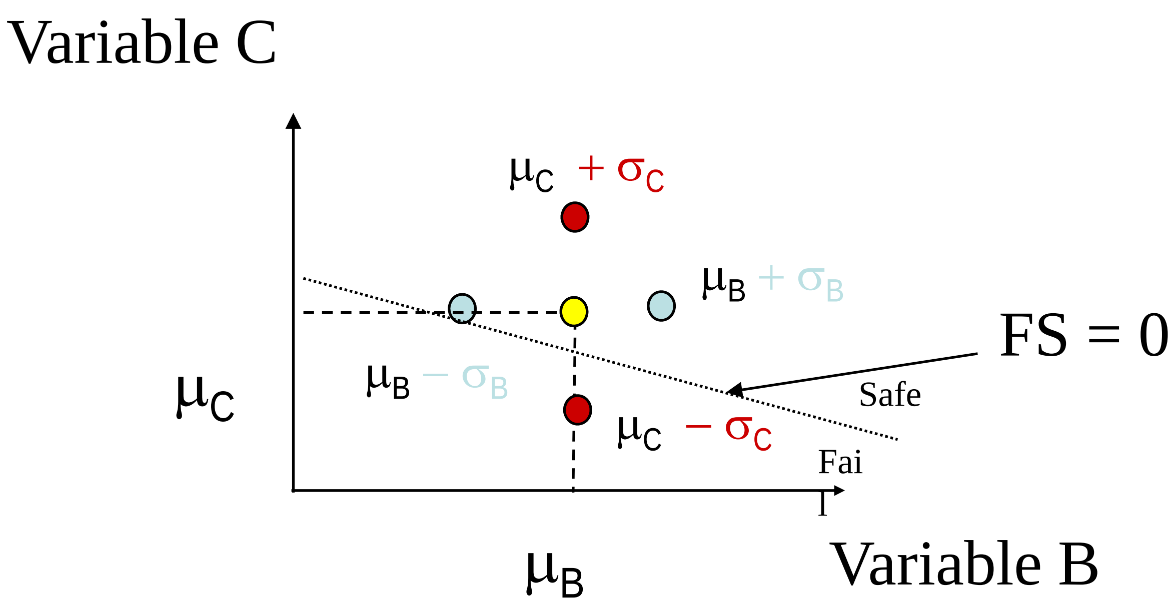
- Reliability Index (β) Methods

  – Taylor Series Finite Difference

  (Cornell, 1969 and Rosenblueth, 1972)

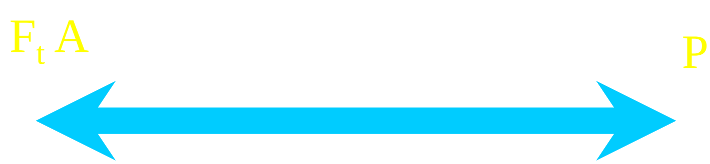
  - First-order expansion about mean value
  - **Linear** approximation of second moment
  - Uses analytical equations (deflection, moment, stress/strain, etc…)
  - Easy to implement in spreadsheets
  - Requires 2n+1 sampling (n = number of variables)
  - Results in a Reliability Index value (β)
    – Based on E(SM) and σ (SM)
  - Problem: caution should be exercised on non-linear limit states

Taylor Series Finite Difference



Reliability Example

- Determine the reliability of a tension bar using the TSFD reliability index (β) method



Limit State = $F_t A / P$

Reliability Example

- Random Variables

  - Ultimate tensile strength, F_t
    - mean, μ = 40 ksi; standard deviation, σ = 4 ksi
    - assume normal distribution
  - Load, P
    - mean, μ = 15 kips; standard deviation, σ = 3 kips,
    - assume normal distribution
  - Area, A
    - constant (no degradation) circular cross section, A = 0.5 in^2

Reliability Example

- Mean FS
  - μ_{FS} = 40(0.50)/15 = 1.333

- Standard Deviation FS
  - Ft FS+ = 44 (0.5)/15 = 1.467 FS- = 36 (0.5)/15 = 1.20
  - P FS+ = 40 (0.5)/18 = 1.111 FS- = 40 (0.5)/12 = 1.667
  - $\sigma_{FS} = ([(1.467 - 1.200) / 2]^2 + [(1.111 - 1.667) / 2]^2)^{1/2}$
  - $\sigma_{FS} = (0.134^2 + 0.278^2)^{1/2}$
  - $\sigma_{FS} = 0.309$

Reliability Example

- Reliability Index

β = E[SM]-1/ σ[SM] = 0.333/ 0.309

- β = 1.06
- P(u) = 0.14
- R = 1 – P(u) = 0.86

- Reliability Index (β) Methods
- Point Estimate Method

  (Rosenblueth (1975))
  - Based on analytical equations like TSFD
  - Quadrature Method
  - Finds the change in performance function for all combinations of random variable, either plus or minus one standard deviation
    - For 2 random variables - ++, +-, -+, -- (+ or – is a standard deviation)
  - Requires 2^n samplings (n = number of random variables)
  - Results in a Reliability Index value (β)
    - Based on E(SM) and σ (SM)

Point Estimate Method

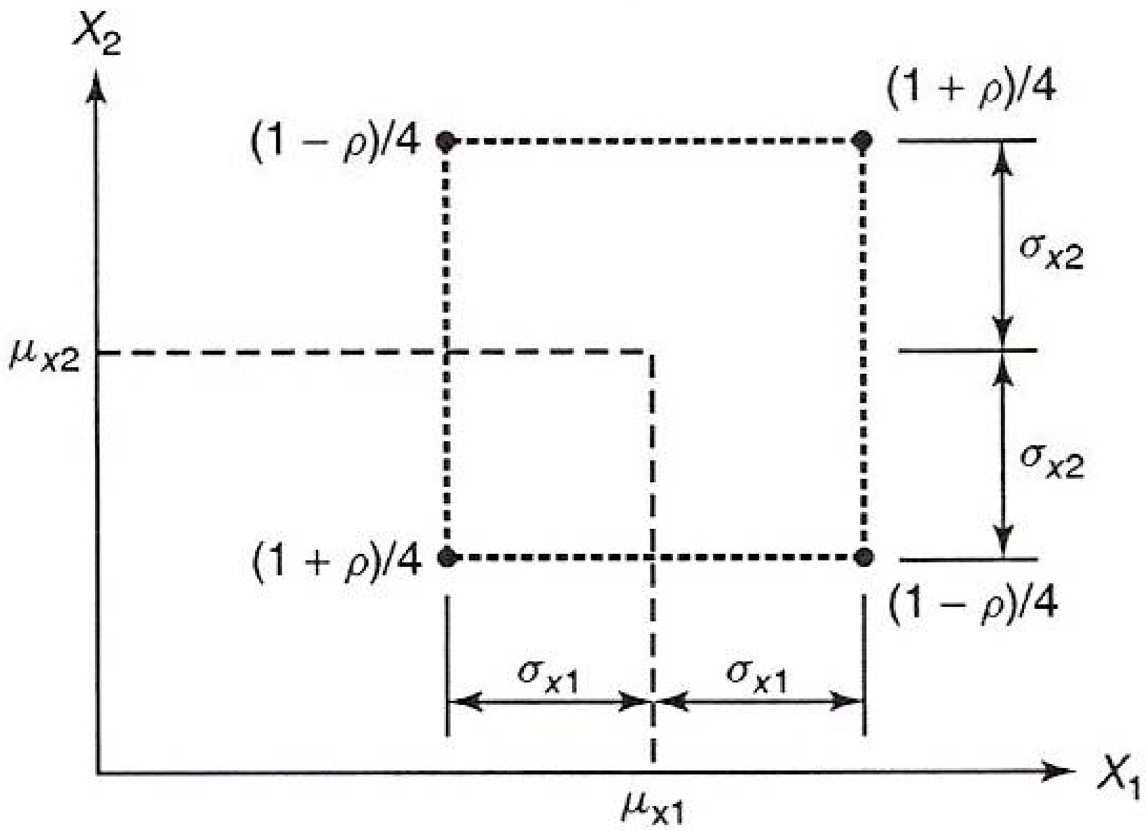


Figure from Baecher and Christian (2003)

Point Estimate Method

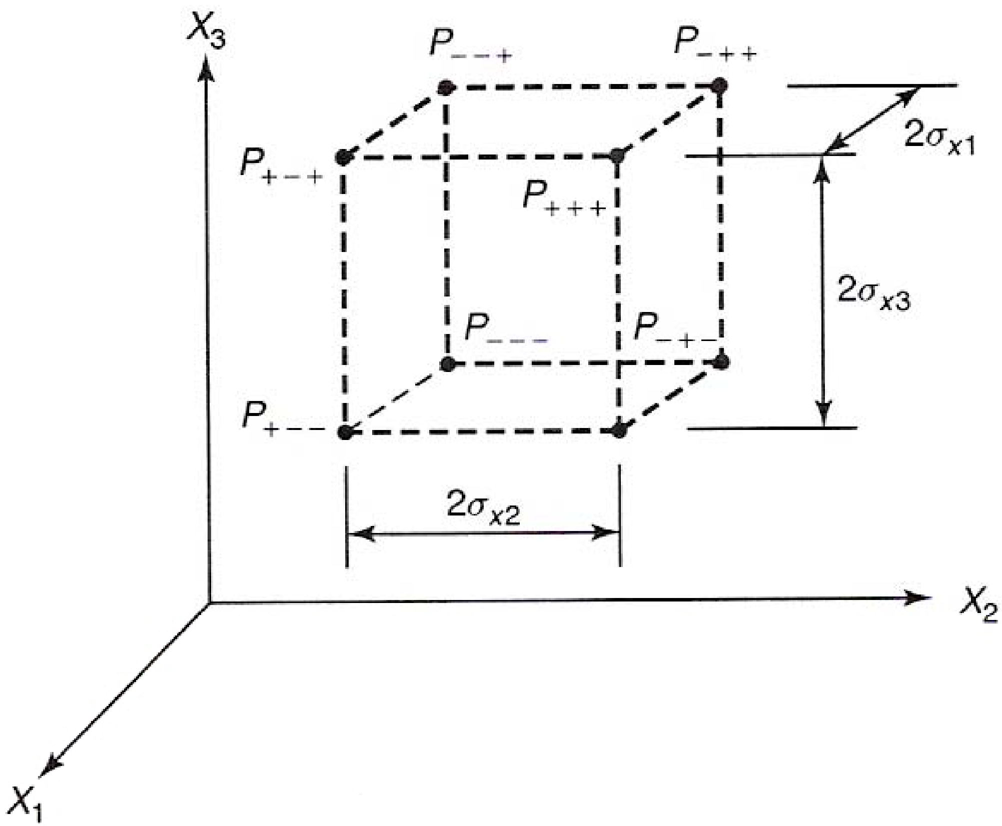
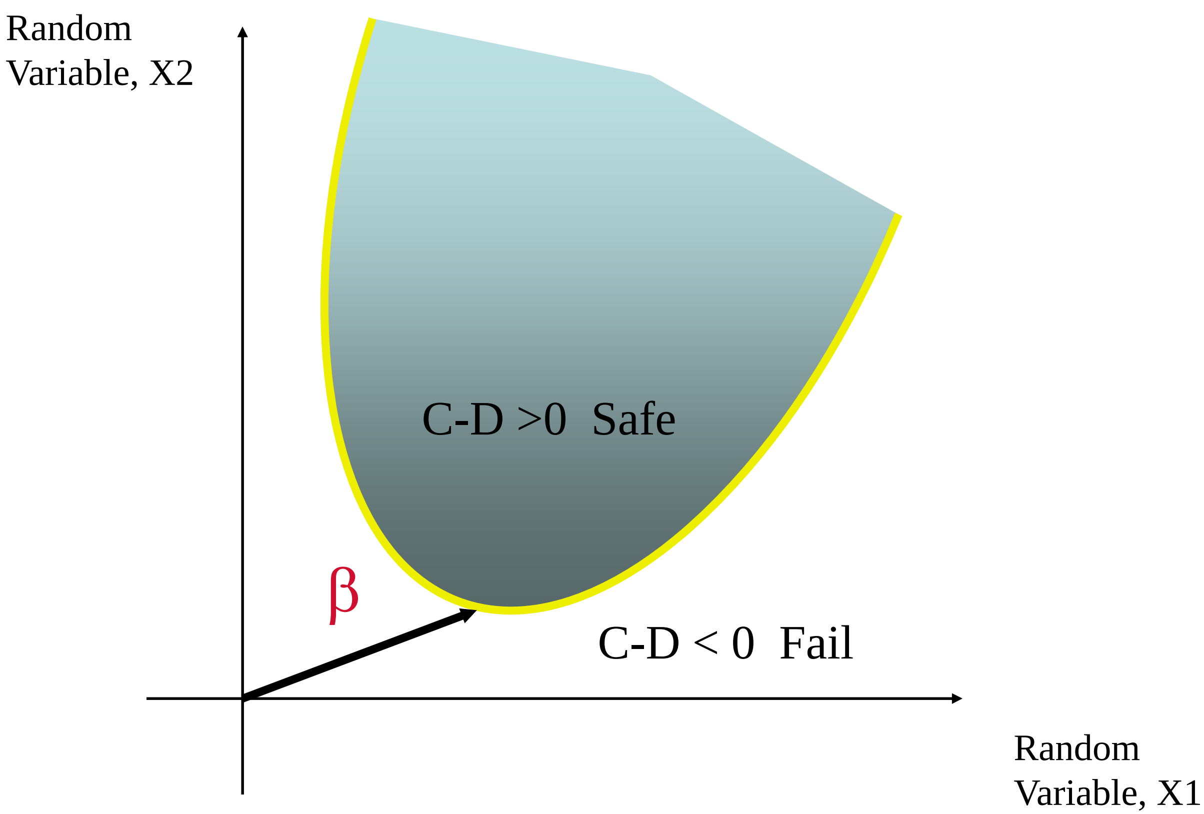


Figure from Baecher and Christian (2003)

- Reliability Index (β) Methods
  - Advanced Second Moment
    (Hasofer-Lind 1974, Haldar and Ayyub 1984)
    - Based on analytical equations like PEM
    - Uses directional cosines to determine shortest distance (β) to multi-dimensional failure surface
    - Accurate for non-linear limit states
    - Solved in spreadsheets or computer programs

Reliability



- Reliability Index (β) Methods

- Shortcomings
  - Instantaneous - capture a certain point in time
  - Index methods cannot be used for time-dependent reliability or to estimate hazard functions even if fit to Weibull or similar distributions
  - Incorrect assumptions are sometimes made on underlying distributions to use β to estimate the probability of failure

- <u>Monte Carlo Simulation</u>
  - "Monte Carlo" is the method (code name) for simulations relating to development of atomic bomb during WWII
    - Traditional – static not dynamic (not involve time), U(0,1)
    - Non-Traditional – multi-integral problems, dynamic (time)
  - Applied to wide variety of complex problems involving random behavior
  - Procedure that generates values of a random variable based on one or more probability distributions
  - Not simulation method per se – just a name!

- <u>Monte Carlo Simulation</u>
  - Usage in USACE
    - Development of numerous state-of-the-art USACE reliability models (structural, geotechnical, etc..)
    - Used with analytical equations and other advanced reliability techniques
    - Determines P_f directly using output distribution
    - Convergence must be monitored
      - Variance recommend

- Monte Carlo Simulation
  - Reliability
    - Determined using actual distribution or using the equation:

$$R = 1 - P(u)$$

where, $P(u) = N_{pu} / N$

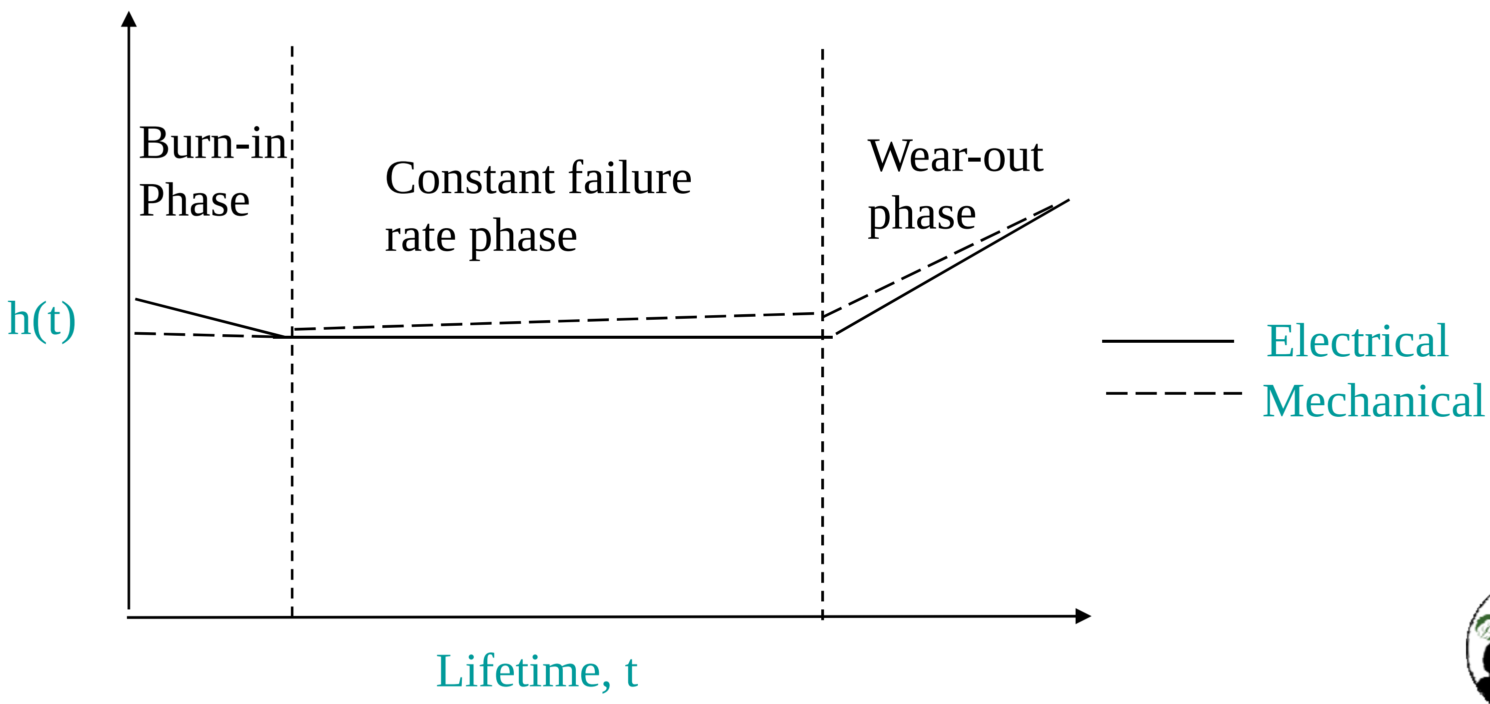
N_{pu} = Number of unsatisfactory performances at limit state < 1.0

N = number of iterations

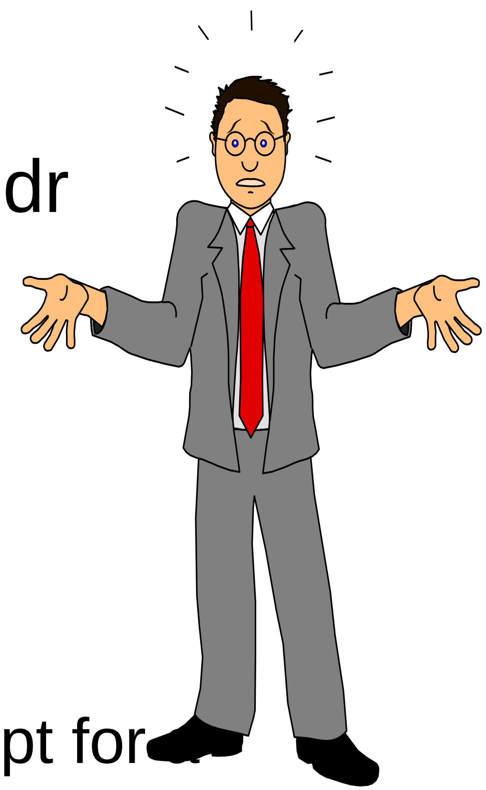
- <u>Hazard Functions</u>
  - <u>Background</u>
    - Previously used reliability index (β) methods
    - Good estimate of relative reliability
    - Easy to implement
    - Problem: “Instantaneous” - snapshot in time

- Hazard Functions/Rates
  - Started with insurance actuaries in England in late 1800's
    - They used the term mortality rate or force of mortality
  - Brought into engineering by the Aerospace industry in 1950's
  - Accounts for the knowledge of the past history of the component
  - Basically it is the rate of change at which the probability of failure changes over a time step
  - Hazard function analysis is not snapshot a time (truly cumulative)
    - Utilizes Monte Carlo Simulation to calculate the true probability of failure (no approximations)
  - Easy to develop time-dependent and non-time dependent models from deterministic engineering design problems

- Typical Hazard Bathtub Curve

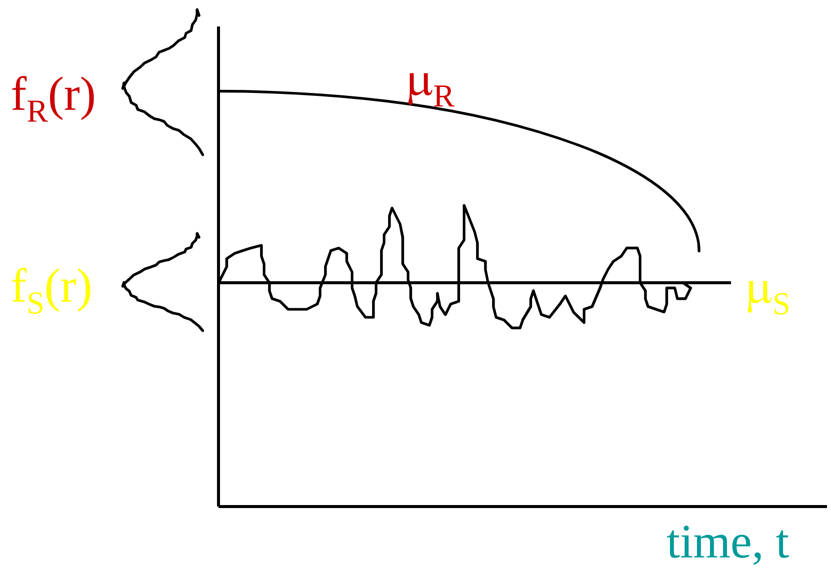


- Ellingwood and Mori (1993)

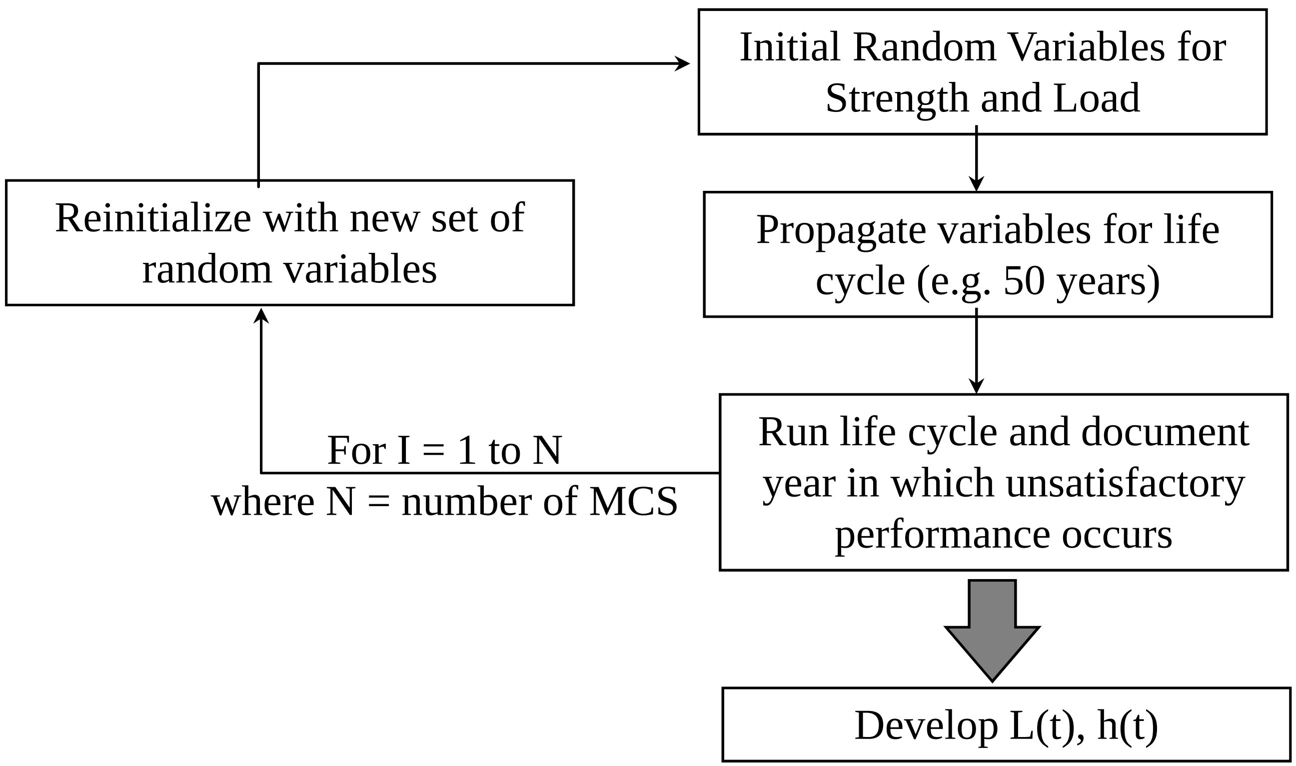


  - $L(t) = \int_0^{\infty} \exp\left[-\lambda t\left[1-1/t \int_0^{t} F_S(g(t)r)\, dt\right]\right] f_R(r)dr$
    - F_S = CDF of load
    - $g(t)r$ = time-dependent degradation
    - $f_R(r)dr$ = pdf of initial strength
    - □ λ = mean rate of occurrence of loading
  - Closed-form solutions are not available except for a few cases
  - Solution: Utilize monte carlo simulations to examine the “life cycle” for a component or structure

- <u>Hazard Functions</u>
  - <u>Degradation of Structures</u>
    - Relationship of strength (R) (capacity) vs. load (S) (demand)

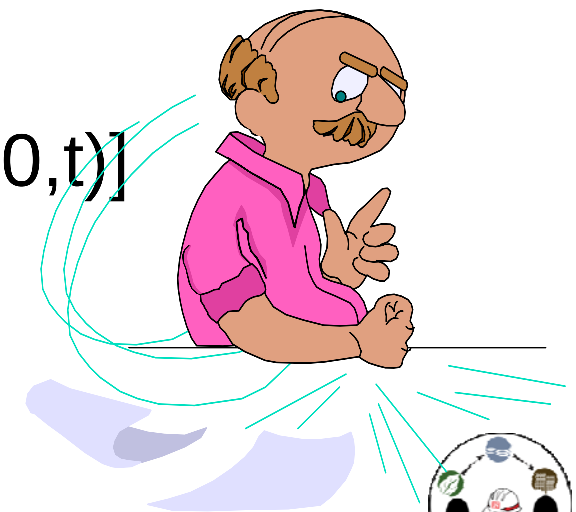


- <u>Life Cycle</u>



- <u>Hazard Function (conditional failure rate)</u>
  - Developed for the ORMSSS economists/planners to assist in performing their economic simulation analysis for ORMSSS investment decisions
  - $h(t) = P[\text{fail in } (t, t+dt) \mid \text{survived } (0,t)]$
  - $h(t) = f(t) / L(t)$

$$= \frac{\text{No. of failures in } t}{\text{No. of survivors up to } t}$$



- Response Surface Methodology (RSM)
  - Reliability is expressed as a limit state function, Z which can be a function of random variables, X_n, where

$$Z = g\ (X_1, X_2, X_3, \ldots)$$

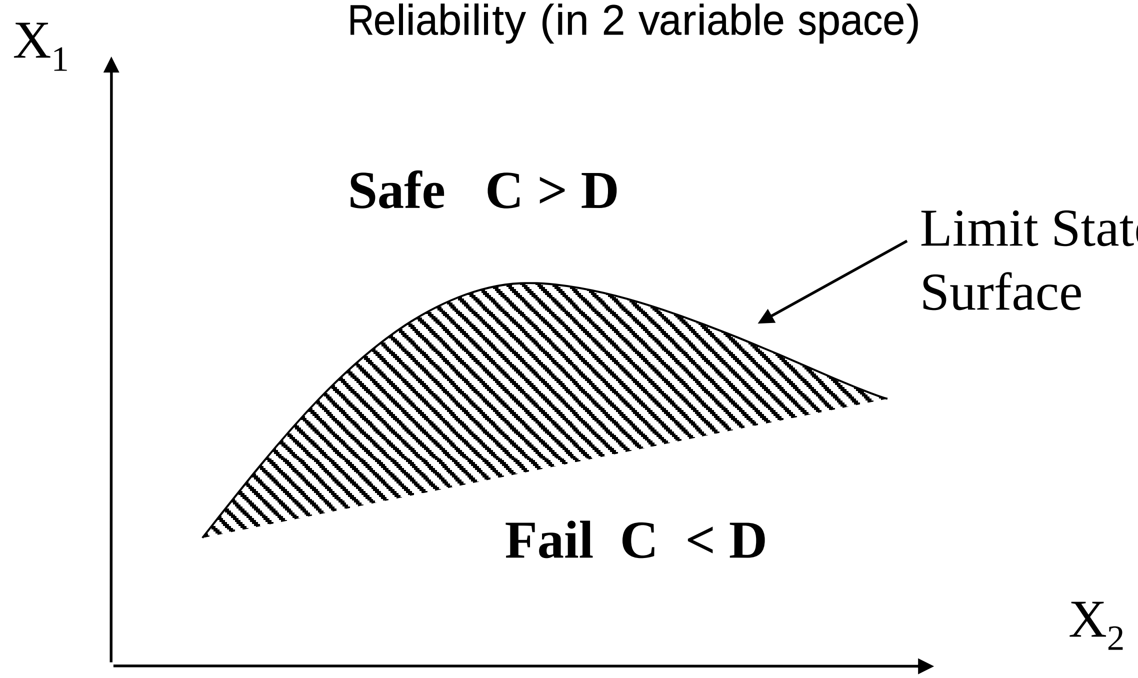
and the limit state is expressed as

$$Z = C - D > 0$$

where D is demand and C is capacity

Response Surface Methodology (RSM)

Reliability (in 2 variable space)



Response Surface Methodology (RSM)

– Utilizes non-linear finite element analysis to define to the response surface

– Not closed form solution but close approximation

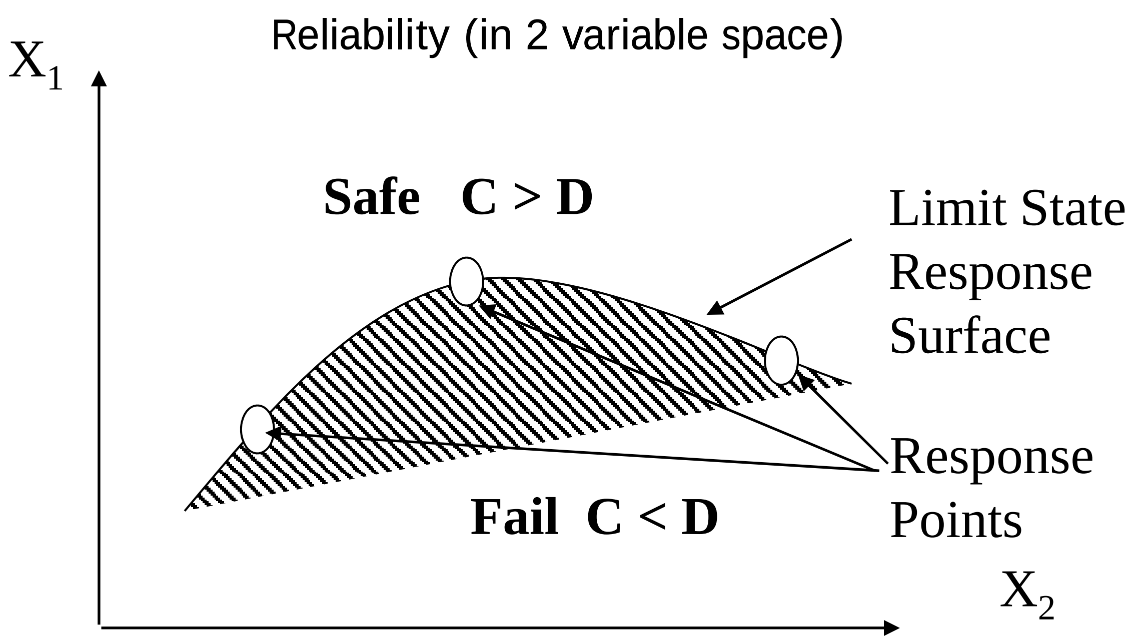
– Constitutive models generally not readily available for performance limit states

  - Typical design equations generally are not adequate to represent limit state for performance

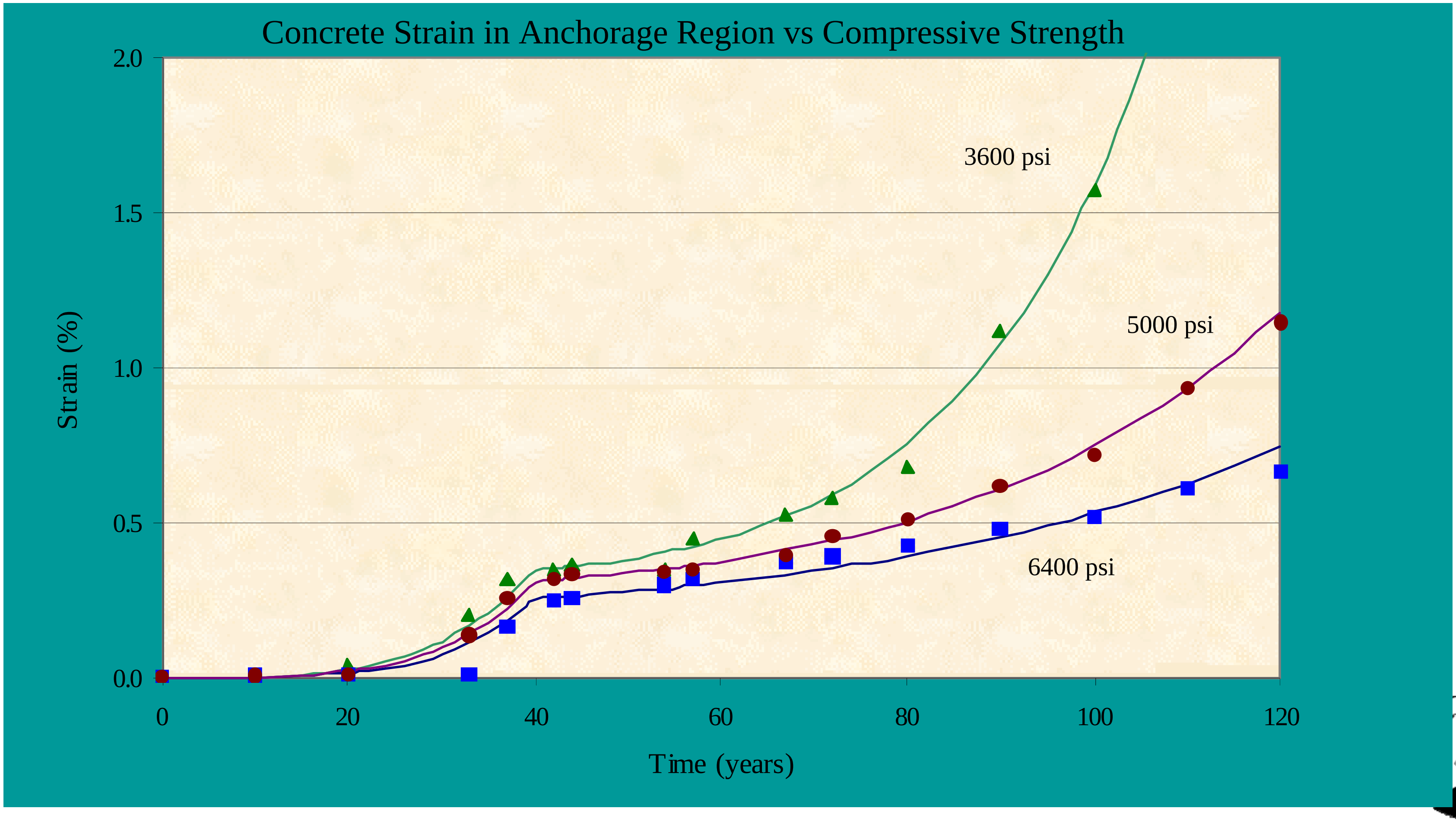
Response Surface Methodology (RSM)

- Accounts for variations of random variables on response surface
- Reflects realistic stresses/strains, etc. that are found in navigation structures
- Calibrated to field observations/measurements
- Develop response surface equations and use Monte Carlo Simulation to perform the reliability calculations
- Recent USACE Applications
  - Miter Gates (welded and riveted)
  - Tainter Gates
  - Tainter Valves (horizontally and vertically framed)
  - Alkali-Aggregate Reaction

Response Surface Methodology (RSM)

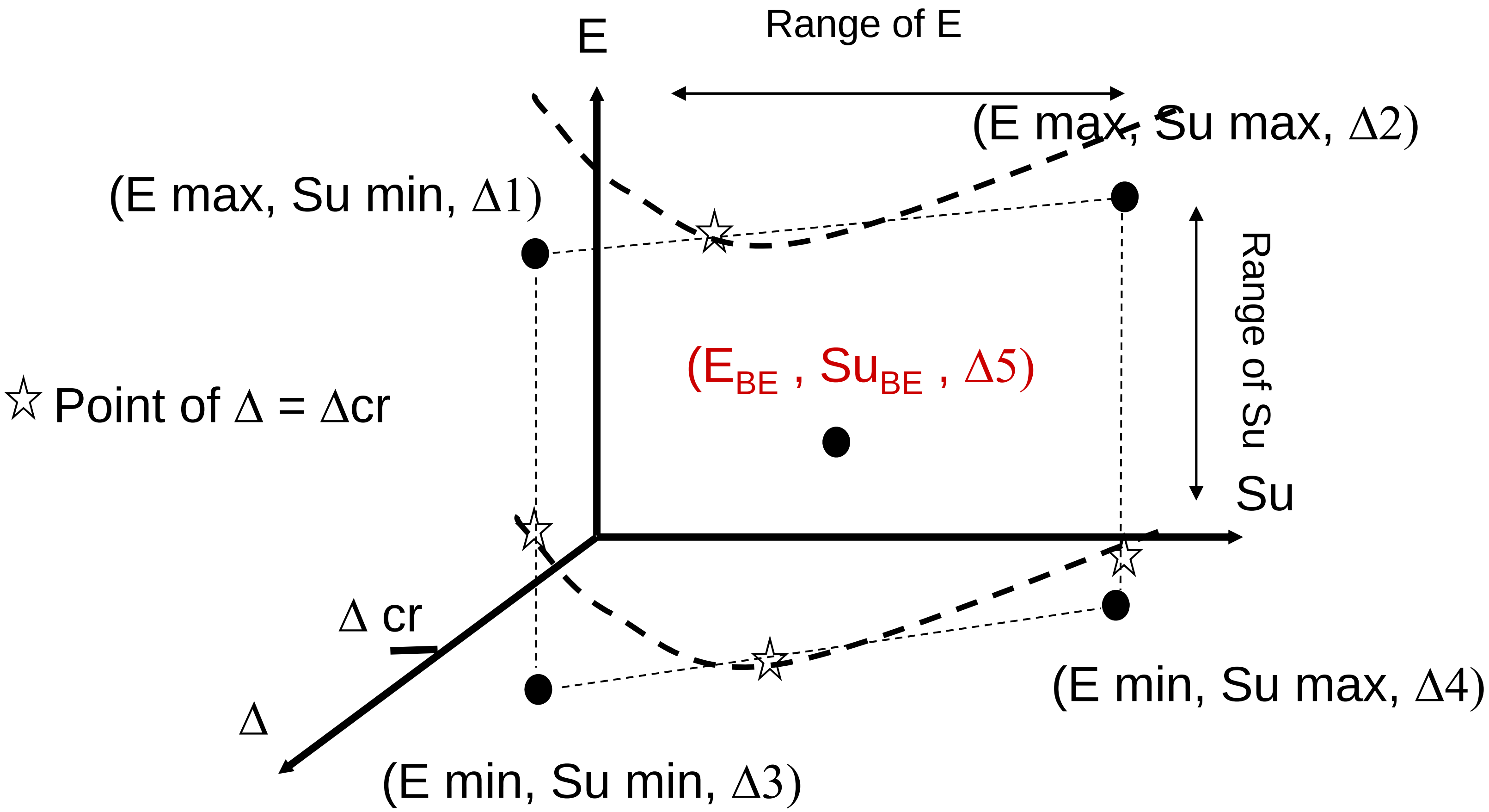


Response Surfaces



- Response Surface Methodology
  - Proposed Methodology for I-Wall Reliability
    - Assumptions
      - Poisson ratio – constant
      - Random variables – E, Su (G, K)
    - Limit state based on deflection (Δ) at ground surface
    - $g(\Delta) = f(E, Su) = \Delta_{cr} / \Delta < 1.0$

Response Surface Modeling Concept (under development)



Reliability

- **Preferred Methods**
  - For non-time dependent reliability problems
    - Linear – Taylor Series Finite Difference, Point Estimate or Monte Carlo Simulation
      - Assume normal distributions for TSFD
      - Assume any distributions for MCS
    - Non-Linear – Advanced Second Moment or Monte Carlo Simulation
  - For time-dependent reliability problems
    - Hazard Function/Rates using Monte Carlo Simulation